

Two analytical constraints on the η - η' mixing

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We obtained two analytical constraints on the η - η' mixing parameters by considering two-photon decays of η and η' [$\eta(\eta') \rightarrow \gamma\gamma$], and productions of η and η' in the e^+e^- scattering at large momentum transfer ($Q^2 \rightarrow \infty$). Using the data given in the PDG98 for the decay processes and recent CLEO measurements on the meson-photon transition form factors, we estimate for the η_8 - η_1 mixing scheme the mixing angle to be $\theta = -14.5^\circ \pm 2.0^\circ$ and the ratio of the decay constants of singlet to octet to be $f_1/f_8 = 1.17 \pm 0.08$. Applying our approach to the recently proposed $q\bar{q}$ - $s\bar{s}$ mixing scheme, we obtain the mixing angle to be $\phi = 39.8^\circ \pm 1.8^\circ$ and the ratio of the decay constants of $s\bar{s}$ state to $q\bar{q}$ state to be $f_s/f_q = 1.20 \pm 0.10$.

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As is well known, the $SU(3)$ quark model predicts the existence of an octet of massless pseudoscalar $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and a massive singlet $\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$. The physical states η and η' arise from the mixing between η_8 and η_1 ,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}. \quad (1)$$

Studying the η - η' mixing is important to the understanding of quark model and QCD. There have been many studies on the mixing angle θ , see e.g. [1–14] and references therein. The recent experimental measurements on the productions of η and η' from the CLEO collaboration [15] and L3 collaboration [16] have stimulated more phenomenological studies on this issue [11–14]. The theoretical analyses are usually model dependent, and the predictions for θ vary from -12° [4,13] to -20° [2,9]. Besides the mixing angle θ , another two parameters f_8 and f_1 , the decay constants of η_8 and η_1 , are usually introduced. Model analyses estimate f_8 in the range of $0.71f_\pi$ [5]³ to $1.28f_\pi$ [2,8,12,13] and f_1 in the range of $0.94f_\pi$ [5] to $1.25f_\pi$ [8]. Once again, the uncertainty in the model predictions is sizable.

Recently a two-mixing-angle scheme [4,5,8,12] was investigated, especially in the studies of the mixing of decay constants,

$$\begin{pmatrix} f_\eta \\ f_{\eta'} \end{pmatrix} = \begin{pmatrix} \cos \theta_8 & -\sin \theta_1 \\ \sin \theta_8 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} f_{\eta_8} \\ f_{\eta_1} \end{pmatrix}. \quad (2)$$

Although it was frequently assumed [2,3,6,7,9,14] that the decay constants follow the pattern of state mixing [Eq. (1)], the mixing properties of the decay constants will be generally different from the mixing properties of the meson state since the decay constants are controlled by the specific Fock state wave functions at zero spatial separation of the quarks while the state mixing refers to the mixing of the overall wave functions [13]. The two-angle-mixing scheme, Eq. (2), is suitable in the studies of the mixing of decay constants, while the common η_8 - η_1 mixing scheme, Eq. (1), is applicable in the studies of state mixing since only one mixing

³We quoted the value evaluated from [5] by the authors of Ref. [13].

angle is required in this situation. More recently a mixing scheme based on the quark flavour basis $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ was proposed [13]. It was assumed that the decay constants follow the state mixing if and only if they are defined with respect to the $q\bar{q}$ - $s\bar{s}$ basis,

$$\begin{pmatrix} \eta(f_\eta) \\ \eta'(f_{\eta'}) \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q(f_{\eta_q}) \\ \eta_s(f_{\eta_s}) \end{pmatrix}, \quad (3)$$

where ϕ is the mixing angle. One advantage of the $q\bar{q}$ - $s\bar{s}$ mixing scheme is that only one mixing angle is introduced but not two as that in Eq. (2).

In this letter, we concern ourselves about the mixing of particle states [the mixing of decay amplitudes (see Eqs. (7) and (8)) and the mixing of meson-photon transition form factors (see Eqs. (17) and (18))] but not the mixing of decay constants. Thus we employ the η_8 - η_1 mixing scheme [Eq. (1)] and the $q\bar{q}$ - $s\bar{s}$ mixing scheme [Eq. (3)]. We emphasize that in our analyses the decay constants are not implied to follow the same mixing schemes as the particle states, *i. e.* Eqs. (1) and (3) will not be applied to the mixing of the decay constants. It has been well known [10,12,14] that the two-photon decays of η and η' and the production processes of η and η' in the e^+e^- scattering can be used to extract the mixing parameters. However, we will analyse these exclusive processes in a different way from that in Refs. [10,12,14]: Considering the ratio of the two-photon decay widths of η and η' and the ratio of the η - γ and η' - γ transition factors at the large momentum transfer limit ($Q^2 \rightarrow \infty$), we find that the above ratios satisfy two analytical equations which involved the same two mixing parameters – the mixing angle and the ratio of decay constants. From the two equations we obtain analytical expressions for the mixing angle (θ or ϕ) and the ratio of the decay constants (f_1/f_8 or f_s/f_q) for the η_8 - η_1 mixing and $q\bar{q}$ - $s\bar{s}$ mixing schemes respectively.

The first constraint comes from the two-photon decays of η and η' . The decay amplitudes of $\pi^0 \rightarrow \gamma\gamma$, $\eta_8 \rightarrow \gamma\gamma$ and $\eta_1 \rightarrow \gamma\gamma$ have the same Lorentz structure [17],

$$A_{P \rightarrow r(k_1)r(k_2)} = \frac{\alpha}{\pi} \frac{c_P}{f_P} \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu^*(k_1) \epsilon_\nu^*(k_2) k_{1\alpha} k_{2\beta}, \quad (4)$$

where

$$c_P = \left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right) \quad (5)$$

for the unmixed states $P = (\pi^0, \eta_8, \eta_1)$ and f_P is the corresponding decay constant ($f_\pi = 93$ MeV). Generalizing the PCAC result for $\pi^0 \rightarrow \gamma\gamma$ to the decays $\eta_8 \rightarrow \gamma\gamma$ and $\eta_1 \rightarrow \gamma\gamma$, and assuming the mixing occurs at amplitude level [2], we can obtain

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = \frac{\alpha^2 m_{\pi^0}^3}{64 \pi^3} \left[\frac{c_{\pi^0}}{f_\pi} \right]^2, \quad (6)$$

$$\Gamma_{\eta \rightarrow \gamma\gamma} = \frac{\alpha^2 m_\eta^3}{64 \pi^3} \left[\frac{c_8 \cos \theta}{f_8} - \frac{c_1 \sin \theta}{f_1} \right]^2, \quad (7)$$

$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{\alpha^2 m_{\eta'}^3}{64 \pi^3} \left[\frac{c_8 \sin \theta}{f_8} + \frac{c_1 \cos \theta}{f_1} \right]^2. \quad (8)$$

Combining Eqs. (7) and (8) yields

$$\frac{\Gamma_{\eta \rightarrow \gamma\gamma}}{\Gamma_{\eta' \rightarrow \gamma\gamma}} = \frac{m_\eta^3}{m_{\eta'}^3} \left[\frac{f_1/f_8 - c_1/c_8 \tan \theta}{f_1/f_8 \tan \theta + c_1/c_8} \right]^2. \quad (9)$$

It is worth noting that only two parameters, θ and f_1/f_8 , appear in Eq. (9) while three parameters, θ , f_1 and f_8 , are involved in Eqs. (7) and (8). If we now let

$$c = \frac{c_1}{c_8} = \sqrt{8}, \quad (10)$$

$$r = \frac{f_1}{f_8}, \quad (11)$$

$$\rho_1 = \left[\frac{\Gamma_{\eta \rightarrow \gamma\gamma}}{\Gamma_{\eta' \rightarrow \gamma\gamma}} \frac{m_{\eta'}^3}{m_\eta^3} \right]^{1/2}, \quad (12)$$

we obtain the first constraint on the parameters θ and r from Eq. (9)

$$\tan \theta = \frac{r - c \rho_1}{c + r \rho_1}. \quad (13)$$

The second constraint comes from the productions of η and η' in the e^+e^- scattering, $e^+e^- \rightarrow e^+e^-\eta(\eta')$ [$\gamma\gamma^* \rightarrow \eta(\eta')$], at large momentum transfer. This class of process can be described using only one form factor, the meson-photon transition form factor $F_{P\gamma}(Q^2)$. It has been noted [10,12,14] that this form factor may provide useful information on the η - η' mixing. The common procedure to extract the η - η' mixing angle from these processes is to fit the perturbative calculations for the transition form factors to the experimental data in

the range of Q^2 being larger than, say, 1 or 2 GeV². At present the available experimental data for the $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ are in the ranges of $Q^2 < 20$ GeV² and $Q^2 < 30$ GeV², respectively [15,16]. As we know, although perturbative theory can make reliable prediction for the asymptotic ($Q^2 \rightarrow \infty$) behavior of exclusive processes, the perturbative calculations in the currently experimentally accessible energy region, especially in the lower energy end of the experimental data, may suffer from large corrections such as higher order contributions in $\alpha_s(Q^2)$ and higher twist effects. Also the perturbative calculations usually employ some model wave functions to account for the non-perturbative properties of η and η' [10,12,14]. Thus the usual procedure to extract the η - η' mixing angle from these production processes has large uncertainty, and even the reliability of this procedure was questioned in Ref. [14] by studying these form factors in light-cone perturbative theory. Here we would like to adopt an alternative scheme: combining the well established perturbative predictions for the η and η' transition form factors in the asymptotic limit ($Q^2 \rightarrow \infty$) with the phenomenological formulas given by the CLEO collaboration and L3 collaboration to obtain another analytical constraint on the mixing angle θ and the ratio f_1/f_8 .

The $Q^2 \rightarrow \infty$ behavior of $F_{P\gamma}(Q^2)$ is well predicted by perturbative QCD [18,19]. For the π^0 , η_8 , and η_1 , we have

$$F_{P\gamma}(Q^2) = \frac{2}{\sqrt{3}} c_P \int_0^1 dx \frac{\phi_P(x)}{x(1-x)Q^2}, \quad (14)$$

where x and $1-x$ are the longitudinal momentum fractions carried by the quark and anti-quark in the meson respectively, and c_P is given by Eq. (5). In the $Q^2 \rightarrow \infty$ limit, any meson distribution amplitudes approach the asymptotic form

$$\phi_P(x) = \sqrt{3} f_P x(1-x). \quad (15)$$

Thus we have

$$F_{P\gamma}(Q^2 \rightarrow \infty) = \frac{2c_P f_P}{Q^2}. \quad (16)$$

Assuming the mixing occurs at the states, we have

$$F_{\eta\gamma}(Q^2 \rightarrow \infty) = \frac{2c_8 f_8}{Q^2} \cos \theta - \frac{2c_1 f_1}{Q^2} \sin \theta, \quad (17)$$

$$F_{\eta'\gamma}(Q^2 \rightarrow \infty) = \frac{2c_8 f_8}{Q^2} \sin \theta + \frac{2c_1 f_1}{Q^2} \cos \theta. \quad (18)$$

If we let

$$\rho_2 = \frac{F_{\eta\gamma}(Q^2 \rightarrow \infty)}{F_{\eta'\gamma}(Q^2 \rightarrow \infty)}, \quad (19)$$

we can obtain the second constraint on the mixing angle θ and the ratio f_1/f_8 from Eqs. (17) and (18),

$$\tan \theta = \frac{1 - c r \rho_2}{c r + \rho_2}. \quad (20)$$

We would like to emphasize that Eq. (20) is an analytical expression which depends on the experimental information on the ratio of transition form factors $F_{\eta\gamma}(Q^2)/F_{\eta'\gamma}(Q^2)$ at large momentum transfer. Eq. (20) suffers from experimental uncertainties on the measurements of $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ at large (but finite) momentum transfer and the systematic uncertainties associated with the extrapolation of $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ from experimentally accessible energy region to infinity [see Eq. (29)]. The usual procedure to extract the mixing angle from these form factors suffer from large corrections to the perturbative calculations in the energy region of a few GeV^2 as well as the experimental uncertainty.

From Eqs. (13) and (20), we obtain the following analytical expressions for the mixing θ and ratio f_1/f_8 ,

$$\tan \theta = \frac{-(1 + c^2)(\rho_1 + \rho_2) + \sqrt{(1 + c^2)^2(\rho_1 + \rho_2)^2 + 4(c^2 - \rho_1 \rho_2)(1 - c^2 \rho_1 \rho_2)}}{2(c^2 - \rho_1 \rho_2)}, \quad (21)$$

$$r = \frac{(1 + c^2)(\rho_1 - \rho_2) + \sqrt{(1 + c^2)^2(\rho_1 - \rho_2)^2 + 4c^2(1 + \rho_1 \rho_2)^2}}{2c(1 + \rho_1 \rho_2)}. \quad (22)$$

From Eqs. (6) and (7), we can also evaluate the ratios f_8/f_π and f_1/f_π ,

$$\frac{f_8}{f_\pi} = \rho_0 \left[\frac{c_8}{c_\pi} \cos \theta - \frac{1}{r} \frac{c_1}{c_\pi} \sin \theta \right], \quad (23)$$

$$\frac{f_1}{f_\pi} = \rho_0 \left[\frac{c_8}{c_\pi} r \cos \theta - \frac{c_1}{c_\pi} \sin \theta \right], \quad (24)$$

where

$$\rho_0 = \left[\frac{\Gamma_{\pi^0 \rightarrow \gamma\gamma}}{\Gamma_{\eta \rightarrow \gamma\gamma}} \frac{m_\eta^3}{m_{\pi^0}^3} \right]^{1/2}. \quad (25)$$

The parameters ρ_0 and ρ_1 [see Eq. (12)] can be fixed by using the two photon decay widths of π^0 , η and η' and their masses. We employ the data given in the PDG98 [20],

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = 7.74 \pm 0.55 \text{eV}, \quad m_{\pi^0} = 134.9764 \pm 0.0006 \text{MeV}, \quad (26)$$

$$\Gamma_{\eta \rightarrow \gamma\gamma} = 0.46 \pm 0.04 \text{keV}, \quad m_\eta = 547.30 \pm 0.12 \text{MeV}, \quad (27)$$

$$\Gamma_{\eta' \rightarrow \gamma\gamma} = 4.37 \pm 0.25 \text{keV}, \quad m_{\eta'} = 957.78 \pm 0.14 \text{MeV}. \quad (28)$$

The parameter ρ_2 [see Eq. (19)] can be determined by using the experimental data on the transition form factors at large momentum transfer. Recently, the CLEO collaboration [15] has measured the $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ in the Q^2 regions up to 20 and 30 GeV^2 respectively, and the L3 collaboration [16] has measured the $F_{\eta'\gamma}(Q^2)$ in the Q^2 range up to 10 GeV^2 . Both CLEO and L3 present their results for the transition form factor by a pole form [15,16]

$$F_{P\gamma}(Q^2) = \frac{1}{4\pi\alpha} \left[\frac{64\pi\Gamma_{P \rightarrow \gamma\gamma}}{m_P^3} \right]^{1/2} \frac{1}{1 + Q^2/\Lambda_P^2}, \quad (29)$$

where Λ_P is the pole mass parameter. Thus we have

$$\rho_2 = \left[\frac{\Gamma_{\eta \rightarrow \gamma\gamma}}{\Gamma_{\eta' \rightarrow \gamma\gamma}} \frac{m_{\eta'}^3}{m_\eta^3} \right]^{1/2} \frac{1 + Q^2/\Lambda_{\eta'}^2}{1 + Q^2/\Lambda_\eta^2} \bigg|_{Q^2 \rightarrow \infty} = \rho_1 \frac{\Lambda_\eta^2}{\Lambda_{\eta'}^2}. \quad (30)$$

The CLEO results for the pole-mass parameters [15] are⁴

$$\Lambda_\eta = 774 \pm 11 \pm 16 \pm 22 \text{MeV}, \quad \Lambda_{\eta'} = 859 \pm 9 \pm 18 \pm 20 \text{MeV}, \quad (31)$$

where the first error represents statistical, the second error is systematic, and the third error comes from the uncertainty in the value of $\Gamma_{\eta(\eta') \rightarrow \gamma\gamma}$. Using the data presented in Eqs. (26), (27), (28) and (31), we obtain

⁴We will not use the result of L3 collaboration since only one pole-mass parameter, $\Lambda_{\eta'}$, was given in [16].

$$\theta = -14.5^\circ \pm 2.0^\circ, \quad f_1/f_8 = 1.17 \pm 0.08, \quad (32)$$

$$f_8/f_\pi = 0.94 \pm 0.07, \quad f_1/f_\pi = 1.09 \pm 0.05, \quad (33)$$

where we have added all errors in quadrature in our calculations.

Our prediction for the mixing angle $\theta = -14.5^\circ \pm 2.0^\circ$ is comparable with the result $\theta \simeq -17^\circ$ given in Refs. [6,7] which are obtained by considering various decay processes, especially the J/Ψ decays. Also our prediction is larger than the prediction $\theta \simeq -23^\circ$ [2,9] based on the Chiral Lagrangian and phenomenological mass formulas (Gell-Mann-Okubo formula etc.). We notice that the mixing of decay constants was assumed to follow the same pattern of the state mixing in [2,9] which is questionable as we mentioned in the introduction. Thus the inconsistency between our prediction for the mixing angle and the prediction from Chiral theory [2,9] is not as serious as it seems to be. We will not compare our prediction with the results from the two-mixing-angle schemes [4,5,8,12] since different mixing schemes are employed. Our result for the ratio f_8/f_π being 0.94 ± 0.07 is smaller than the prediction from Chiral perturbation theory (ChPT) $f_8/f_\pi = 1.28$ [2,8] and most phenomenological analyses $f_8/f_\pi = 1.2 - 1.3$ [1,3,9,12,13], but is larger than the result given in Ref. [5] $f_8/f_\pi = 0.71$. Our prediction for the ratio f_1/f_π being 1.09 ± 0.05 is consistent with most phenomenological analyses being about 1.15 [1,3,9,12,13] and the ChPT prediction $f_1/f_\pi = 1.05 \pm 0.04$ given in [2] but is large than the ChPT prediction $f_1/f_\pi = 1.25$ given in [8]. In the previous studies either the questionable assumption that the decay constants and the particle states share the same mixing scheme [1–3,9] or two mixing-angle scheme [5,8,12,13] is adopted. The relations between the mixing parameters involved in the two-mixing-angle scheme and that appear in our model are remained to be further studied. Thus our comparisons for the decay constants here are just suggestive.

Now we turn to the constraints on the parameters involved in the $q\bar{q}-s\bar{s}$ mixing scheme [see Eq. (3)]. The center assumption in this mixing scheme is that the decay constants follow the state mixing if and only if they are defined with respect to the $q\bar{q}-s\bar{s}$ basis [13]. We would like to point out that our analysis for the $\eta_8-\eta_1$ mixing scheme can be easily applied to the $q\bar{q}-s\bar{s}$ mixing scheme by replacing the parameters $c = c_1/c_8$ and $r = f_1/f_8$ in Eqs. (21) and

(22) with $c' = c_s/c_q = \sqrt{2}/5$ and $r' = f_s/f_q$. Employing the data for the two-photon decay of π^0 , η and η' given in the PDG98 [see Eqs. (26), (27) and (28)] and the CLEO result for the transition form factors [see Eq. (31)], we can obtain

$$\phi = 39.8^\circ \pm 1.8^\circ, \quad f_s/f_q = 1.20 \pm 0.10, \quad (34)$$

$$f_q/f_\pi = 1.06 \pm 0.05, \quad f_s/f_\pi = 1.27 \pm 0.12. \quad (35)$$

Our predictions [Eqs. (34) and (35)] are consistent with the phenomenological results $\phi = 39.3^\circ \pm 1.0^\circ$, $f_q/f_\pi = 1.07 \pm 0.02$ and $f_s/f_\pi = 1.34 \pm 0.06$ given in Ref. [13], which implies that as one concerns about the state mixing the one-mixing angle schemes, the η_8 - η_1 mixing [Eq. (1)] and the $q\bar{q}$ - $s\bar{s}$ mixing [Eq. (3)], are applicable. This consistency also implies that our predictions for the η_8 - η_1 mixing parameters [Eqs. (32) and (33)] are reliable.

In summary, we obtained two analytical constraints on the η - η' mixing parameters by considering the two-photon decays $\eta(\eta') \rightarrow \gamma\gamma$ and the productions of η and η' in the e^+e^- scattering at large momentum transfer ($Q^2 \rightarrow \infty$). One advantage of our analysis is that it can be easily updated with any new experimental data on the decay widths and meson-photon transition form factors. Using the data given in the PDG98 for the decay processes and recent CLEO measurements on the meson-photon transition form factors, we obtain $\theta = -14.5^\circ \pm 2.0^\circ$, $f_1/f_8 = 1.17 \pm 0.08$ for the η_8 - η_1 mixing scheme, and $\phi = 39.8^\circ \pm 1.8^\circ$, $f_s/f_q = 1.20 \pm 0.10$ for the recently proposed $q\bar{q}$ - $s\bar{s}$ mixing scheme.

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